where the left-hand side vector contains the components of forces and moments in the panel coordinate system. Substituting Eq. (20) into Eq. (25) yields

$$\underset{6N\times 1}{\begin{Bmatrix} F \\ \hdashline M \end{Bmatrix}} = \underset{6N\times N}{\begin{bmatrix} B \\ \hdashline C \end{bmatrix}} \underset{N\times N}{[A]^{-1}} \underset{N\times 1}{\{\dot{V}_n\}} \tag{26}$$

For solid fluid interaction problems, it is useful to relate the panel accelerations to node acceleration. For instance, assume that the normal acceleration of the ith panel is related to the translational ($\dot{V}_{ij}$) and angular acceleration ($\dot{R}_{ij}$) of the jth node of the same panel by the following relation

$$\{\dot{V}_n\}_i = [T] \begin{Bmatrix} \dot{V} \\ \dot{R} \end{Bmatrix}_{ij} \tag{27}$$

Then, the associated added mass forces are given by substituting Eq. (27) into Eq. (26). The coefficient matrix of $\{\dot{V}_n\}$ in the resulting equation is the added mass matrix of the system.

V. Conclusions

A comprehensive approach for the treatment of acceleration-dependent fluid forces is presented. The methoid is very convenient for programming for digital computer calculations. The other significant advantages of the appraoch are as follows:

1) The approach gives the added mass distribution in addition to the gross coefficients.

2) The approach is applicable for arbitrary three-dimensional bodies; rigid as well as elastic.

3) The linear and nonlinear terms can be calculated.

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Optical Constants of Propellant-Grade Ammonium Perchlorate

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Introduction

CONTROLLING radiative heating of aluminum in composite propellants has recently been brought to light as a possible mechanism for reducing unwanted agglomeration of aluminum during the combustion of aluminized solid propellants.[1] In order to model the radiative scattering and absorption processes within the composite solid propellant, it is necessary to know the radiative properties of the components of the propellant. This Note reports the optical constants n and k of ammonium perchlorate (AP), one of the most common oxidizer materials used in composite solid propellants.

Previous spectroscopic studies[2,3] have been successful at revealing information about the structure and bonding of pure AP. However, these studies do not indicate the absolute magnitude of the optical constants of AP, but rather only the relative absorptivity. Furthermore, these studies deal with pure AP. No studies in the literature report the optical properties of AP used in composite solid propellants. This AP differs from pure AP in that it has, typically, been coated with the anticaking agent tricalcium phosphate (TCP). This small amount of impurity may have some influence on the optical properties of the AP.

Procedure

There are many ways to determine the optical constants of solids.[4,5] In this study, measurements of the near-normal reflectance were made from thin, plane AP disks, over the range of 2.6 to 9.6 μm. The disks were prepared from AP powder. A dispersion equation curve-fitting technique was then applied to find values of n and k consistent with the near-normal reflectance measurements.

To prepare the disks, 100 mg of AP powder was heated in an oven at a temperature of 120–130°C for 8 h to rid the powder of its moisture content. The dry powder was then placed inside a 1.27 cm diam die and a force of 84.5 kN (19,000 lbf) was applied for 5 min to the AP powder. The stainless steel pressing surfaces of the die were optically smooth to form smooth, specular sample surfaces. In addition, the die was evacuated at 25 mm of Hg during the pressing process to prevent air from being trapped inside the disks. The AP disks thus obtained were slightly milky. They appeared specularly reflecting, although some inhomogeneity due to TCP was evident. Two different AP powder sizes were used to make disks, 20 and 50 μm.

A reference measurement of near normal reflectance was made using a first-surface aluminum mirror. The normal reflectance for aluminum was calculated from the known spectral optical constants for aluminum[6] using

$$R_N = \frac{(n-1)^2 + k^2}{(n+1)^2 + k^2} \tag{1}$$

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The determinations of $R_{N_{AP}}$ was then made on a relative basis using the equation,

$$(R_N)_{AP} = \frac{S_{AP}}{S_{AI}} \times (R_N)_{AI} \tag{2}$$

where S is the pyrometer detector reading.

Dispersion Theory

On the basis of the classical treatment, the wavelength or frequency dependence of the optical constants may be explained by considering the solid as an assembly of oscillators that are set in forced vibration by the radiation[7]. This treatment leads to the following equations for optical constants:

$$n^2 - k^2 = n_1^2 + \sum_i \frac{(N_i e^2 f_i / m\epsilon_0)(\omega_{0i}^2 - \omega^2)}{(\omega_{0i}^2 - \omega^2)^2 + \omega^2\gamma_i^2} \tag{3}$$

$$2nk = \sum_i \frac{(N_i e^2 f_i / m\epsilon_0)\omega\gamma_i}{(\omega_{0i}^2 - \omega^2)^2 + \omega^2\gamma_i^2} \tag{4}$$

In this expression N_i, ω_{0i} and γ_i are the strength, frequency, and line width of the "ith" oscillator, respectively. Once the parameters (N_i, ω_{0i}, and γ_i) are known, Eqs. (3) and (4) can be solved for n and k. Then, the normal reflectance is given by Eq. (1).

Curve Fitting

Once the values of reflectance were measured, a minimization technique was used to obtain an accurate curve fit between the dispersion equation predictions and the measured data by adjusting the values of the parameters N_i, ω_{0i}, and γ_i.

The Nelder-Meed method[8-10] was used to minimize the function F given by

$$F = [(R_N - R_{N_1})^2 + (R_N - R_{N_2})^2]^{1/2} \tag{5}$$

In Eq. (5), R_{N_1} and R_{N_2} are the experimentally measured values of reflectance for disks pressed from 50 and 20 μm AP, respectively, see Fig. 1.

Results and Discussion

The measured reflectance spectrum for the AP is shown in Fig. 1. The best curve fit of the data using the dispersion equations is also shown (solid line). The values of the parameters for this fit are given in Table 1.

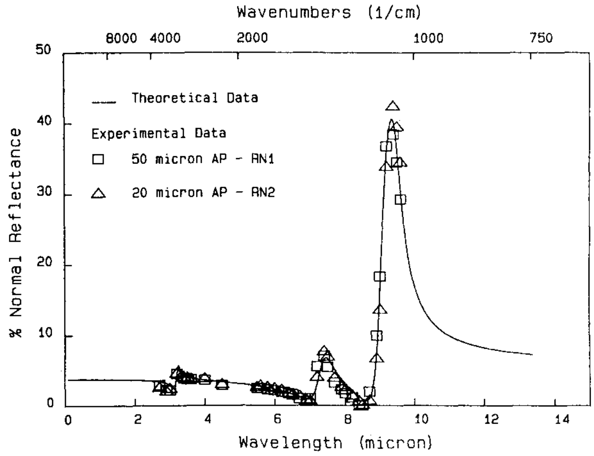

Fig. 1 Spectral reflectance for AP (measured and theory).

Three resonance frequencies were detected in the spectrum at 3.2, 7.35, and 9.45 μm. Relatively small absorption peaks were observed at about 3.2 and 7.35 μm. A strong third peak was observed at about 9.45 μm (corresponding to approximately 40% reflectance).

The optical constants determined are shown in Fig. 2. The main feature of the k spectrum is the strong absorption peak at 9.5 μm, which is attributable to the ClO_4^- distortion.[11] The other two small absorption peaks observed at 3.2 and 7.35 μm are due to N-H stretching and NH_4^+ deformation.[2,3] Two important regions of transmission are indicated by the results for k in Fig. 2, below approximately 2.7 μm and between approximately 3.8 and 4.3 μm. This observation is in good agreement with Fourier transform infrared (FTIR) transmission measurements (Fig. 3) also obtained for the AP samples.

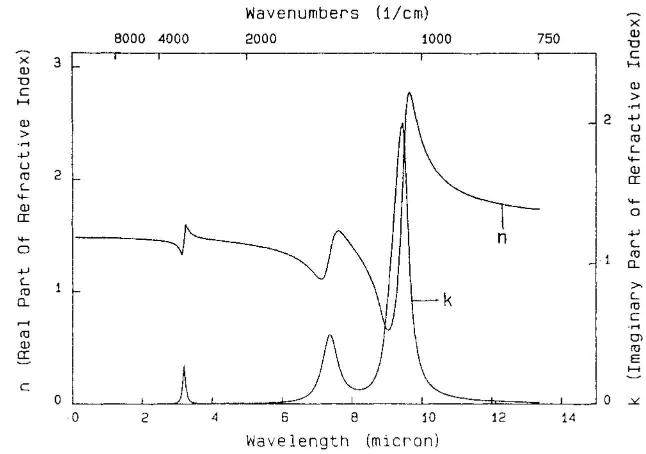

Fig. 2 Real and imaginary parts of refractive index for AP.

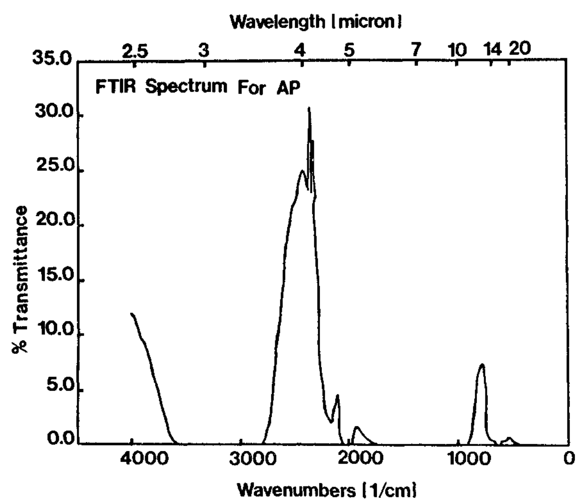

Fig. 3 FTIR transmittance for AP.

Table 1 Parameters of curve fit in Fig. 1

i	N_i, cm^{-3}	ω_{0i}, s^{-1}	γ_i, s^{-1}
1	0.252×10^{19}	58.7×10^{13}	1.66×10^{13}
2	0.188×10^{19}	25.4×10^{13}	1.76×10^{13}
3	0.405×10^{19}	19.8×10^{13}	0.746×10^{13}

Since the main features of the reflectance spectrum can all be assigned to AP, it is concluded that the TCP effect is of second order. This justifies the homogeneous model employed here as opposed to a composite model which might have been used to account for the TCP. It should also be noted that the values for k in the regions of high transmission are very difficult to determine accurately by the present method. This is because it has been assumed that the only resonances present are those observed in the reflectance spectrum and that these resonances have the assumed damped harmonic oscillator form. Deviation from these assumptions would introduce significant errors in the value of k in the very weak absorbing regions.

Conclusions

A dispersion equation curve-fitting technique in conjunction with normal spectral reflectance measurements was used to determine the optical constants of propellant-grade ammonium perchlorate. Although the stringent requirements for sample surface preparation were not satisfied completely the optical constants obtained in this manner represent the only, therefore, the best data available. Furthermore, the values of reflectance calculated from these optical constants are felt to be very representative of as-received propellant-grade AP. Radiative transfer in AP composite propellants is typically dominated by geometric optics scattering (which is dependent on reflectance data and not the precise optical constants). Therefore, the presented results will be very useful for modeling radiative transfer in aluminized composite (AP) propellants in an effort to reduce unwanted aluminum agglomeration caused by radiative heating.

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Identification of Structural Dynamic Systems with Nonproportional Damping

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Introduction

A LINEAR structural dynamic system is usually represented mathematically by a set of second-order linear differential equations. The inertia, damping, and stiffness effects are quantified in terms of mass, damping, and stiffness matrices whose order is equal to the number of degrees of freedom chosen in modeling the physical system. An identification problem involving such systems reduces to the determination of these matrices from a given set of information about the dynamic behavior of the system. For example, the information can be in the form of the system output in the time domain for a specified input. Although a substantial number of techniques have been developed by various researchers to identify undamped or proportionally damped systems, methods capable of handling generalized damping matrices are relatively scarce. Currently existing procedures in this area include the estimation of system matrices using time domain output,[1-3] frequency domain output,[4] and modal parameters.[5,6]

In this paper a different approach to the problem is presented. The technique is based on the assumption that either some or all of the elements of the mass matrix are known. It is also assumed that the complex eigenvalues and the complex eigenvectors of the system have been measured. The remaining elements of the mass matrix, the stiffness matrix, and the damping matrix are determined by minimizing the Euclidean norm of a matrix that assures the satisfaction of the equations of the eigenvalue problem and the appropriate orthogonality conditions. The minimization of the norm has been performed subject to the constraints of symmetry.

Eigenvalue Problem and Identification of System Matrices

The eigenvalue problem for a system with n-degrees-of-freedom can be stated as follows:

$$MU+CV+KW=0 \tag{1}$$

$$U=[\lambda_1^2\phi_1,\lambda_2^2\phi_2\ldots\lambda_n^2\phi_n] \tag{2}$$

$$V=[\lambda_1\phi_1,\lambda_2\phi_2\ldots\lambda_n\phi_n] \tag{3}$$

$$W=[\phi_1,\phi_2\ldots\phi_n] \tag{4}$$

In these equations λ_i and ϕ_i represent the ith eigenvalue and eigenvector, respectively. It is to be noted that in this equa-

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